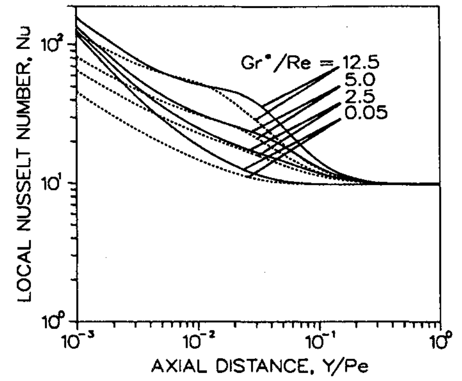

Fig. 4 Effects of Gr^*/Re on the local Nusselt number variation $Da = 0\text{–}10^{-6}$, $Re = 20$ (——), and $Re = 100$ (- - - - -).

value of the Nusselt number increases only at higher Darcy number and remains virtually the same at lower Darcy number.

In the Darcy flow regime it was found that for constant Prandtl number and sufficiently high Reynolds number, heat transfer is governed solely by Gr^*/Re ratio (Fig. 4) instead of Gr^*/Re^2 ratio which is commonly used for mixed convection in nonporous media. This can be verified by an order of magnitude analysis of Eq. (2). In Fig. 4, it is shown that in the inlet and mixed convection region, the Nusselt number increases with increasing Gr^*/Re, and in the fully developed region all the curves converge to the same asymptotic value. However, at moderately low Reynolds number, the axial conduction effects were found to be more significant than in the nonporous channel and Gr^*/Re is no longer the governing parameter as illustrated in Fig. 4.

Conclusions

A numerical study of buoyancy-assisted mixed convection in a vertical porous channel maintained at uniform wall temperatures is performed using the Brinkman-Forchheimer-extended Darcy model.

The results show that when the Darcy number is decreased while the modified Grashof number and the Reynolds number are kept constant, there is a significant increase of the velocities near the walls leading to increased heat transfer, especially in the mixed convection region which starts closer to the channel inlet than in the nonporous channel. In the Darcy flow regime, the governing parameter is Gr^*/Re, but at moderately low Reynolds number, the axial conduction effects (Peclet number effects) are significant.

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Combined Surface Radiation and Free Convection in Cavities

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Nomenclature

A = aspect ratio, H/d
d = spacing, m
F_{ij} = view factor from the ith element to the jth element
g = acceleration due to gravity, m/s^2
H = height of the enclosure, m
J_i = elemental radiosity, W/m^2
j_i = elemental dimensionless radiosity, $J_i/\sigma T_H^4$
k = thermal conductivity of fluid, W/m K
N_{RC} = radiation conduction interaction parameter, $\sigma T_H^4 d/[(T_H - T_c)k]$
Nu_c = local convection Nusselt number, $-(\partial\phi/\partial Y)_{y=0}$
$\overline{Nu_c}$ = mean convection Nusselt number, $\int_0^{2A} Nu_c \, dX/2A$
$\overline{Nu_o}$ = mean overall Nusselt number defined in Eq. (6)
Nu_R = radiation Nusselt number, $[q_{rad}d/k(T_H - T_c)N_{RC}]$
$\overline{Nu_R}$ = mean radiation Nusselt number, $\int_0^{2A} Nu_R \, dX/2A$
Pr = Prandtl number, ν/α
q_{rad} = elemental radiative heat flux based on d, $\varepsilon/(1 - \varepsilon)(\sigma T_i^4 - J_i)$ W/m^2 ($\varepsilon \neq 1$, ε of all elements on a wall is same)
Ra = Rayleigh number based on d, $g\beta(T_H - T_c)d^3/\nu\alpha$
T = temperature, K
T_c = temperature of the right wall of the enclosure, K
T_H = temperature of the left wall of the enclosure, K
T_i = temperature of the ith element on the wall
T_R = temperature ratio, T_c/T_H
U = dimensionless vertical velocity, ud/α
u = vertical velocity, m/s
V = dimensionless horizontal velocity, vd/α
v = horizontal or cross velocity, m/s
W = dimensionless vorticity, wd^2/ν
X = dimensionless vertical coordinate, x/d
x = vertical coordinate, m
Y = dimensionless horizontal coordinate, y/d
y = horizontal coordinate, m
α = thermal diffusivity of fluid, m^2/s

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β = cubical expansion coefficient of fluid, 1/K
ε_{bot} = emissivity of the bottom wall
ε_c = emissivity of the right wall, cold wall
ε_H = emissivity of the left wall, hot wall
ε_{top} = emissivity of the top wall
ν = kinematic viscosity of the fluid, m²/s
σ = Stefan Boltzmann constant, 5.67×10^{-8} W/m²K⁴
ϕ = dimensionless temperature, $(T - T_c)/(T_H - T_c)$
ψ = dimensionless stream function, ψ'/α
ψ' = stream function, m²/s
ω = vorticity, 1/s

Introduction

ALTHOUGH the problem of free convection in cavities has been widely studied, the related problem of combined radiation and free convection in cavities has not received much attention, particularly when the intervening medium is nonparticipating. In many applications, like the cooling of electronic equipment and passive cooling of buildings and insulation design, radiation plays a significant role.[1] Hence, a detailed numerical study of the combined surface radiation and free convection problem at essentially low or moderate temperature levels for cavities with A in the range of 2–20 is made in the present study. The objective is to give correlations for both convective as well as radiative heat transfer across the cavity. The correlation for radiation is superior to the widely used parallel plate formula in such situations.[2]

Formulation and Method of Solution

Free Convection

The governing equations for steady, two-dimensional, laminar, constant property fluid under the Boussinesq approximation, for a rectangular enclosure with side wall heating and adiabatic top and bottom walls, are well established. In the vorticity stream function formulation, the normalized equations become

$$U\frac{\partial W}{\partial X} + V\frac{\partial W}{\partial Y} = Pr(\nabla^2 W) - Ra\frac{\partial \phi}{\partial Y} \tag{1}$$

$$\nabla^2\psi = -PrW \tag{2}$$

$$\nabla^2\phi = U\frac{\partial \phi}{\partial X} + V\frac{\partial \phi}{\partial Y} \tag{3}$$

The boundary conditions are

$$X = 0, 2A, \text{ for all } Y \quad \psi = 0 \text{ (bottom and top walls)} \tag{4a}$$

$$Y = 0, 2, \text{ for all } X \quad \psi = 0 \text{ (left and right walls)} \tag{4b}$$

$$Y = 0, \text{ for all } X \quad \phi = 1 \text{ (left wall)} \tag{4c}$$

$$Y = 2, \text{ for all } X \quad \phi = 0 \text{ (right wall)} \tag{4d}$$

$$X = 0, 2A, \text{ for all } Y \quad \frac{\partial \phi}{\partial X} = 0 \text{ (top and bottom walls)} \tag{4e}$$

Regarding the boundary conditions for vorticity on the solid walls, it is first assumed that the stream function equation [Eq. (2)] is satisfied on the walls. Then the assumption is made that gradients parallel to a particular wall can be neglected in comparison with gradients perpendicular to that wall. This widely used procedure (e.g., Gosman et al.[3]) yields the boundary conditions on vorticity. The above equations were solved using a finite volume method[3] with a 21 × 21 nonuniform cosine grid. With a 21 × 21 nonuniform cosine grid, the first grid spacing is only 0.125 times the grid spacing that would have been obtained with a 21 × 21 uniform grid. However, to test for grid independence, for a typical case of $Ra = 10^6$ and $A = 4$, the equations were solved with a 21 × 21 and 31 × 31 (both nonuniform grids), and the difference in the mean Nusselt numbers was 1.5%. The maximum error in temperature at any of the 441 nodes inside the cavity was found to be 0.8% at $Ra = 10^6$, between two successive iterations when the number of iterations were 600. Stated more explicitly, the convergence on temperature even at the highest Ra was 0.8%. In the Prandtl number range covered in the present study ($0.2 < Pr < 20$), Pr did not have any significant effect on heat transfer. However, it needs to be emphasized that only surface radiation is considered in the present study, and therefore, the radiation calculations are strictly valid for fluids which are transparent. Also, for the range of Pr, Ra, and A used in the present study, it is fairly well established that the multicellular pattern does not significantly affect the average heat transfer.[4]

The error in assuming constant properties for the fluid has been studied by Zhong et al.[5] The error is a strong function of T_R and Ra. From the study of Zhong et al. it is concluded that for the temperature ranges and Rayleigh numbers encountered in the present study, the error incurred in using the Boussinesq approximation is less than 5% on the average Nusselt number, and hence, the maximum error in the present work due to the constant property assumption is 5%. Typically, at $T_R = 0.8$, Zhong et al. obtained an error of only 2% on the Nusselt number by using the Boussinesq approximation.

The adiabatic boundary conditions on the top and bottom walls do not take into account the effect of radiation. This may be justified as follows. In many applications like cooling of electronic equipment, solar collectors, and double pane windows, the T_R varies between 0.75–0.95. This means that both the temperature differentials as well as the absolute values of the temperatures themselves are in the range 0–100°C. Even under the severe case of ε—of all walls being close to 1 at such temperature levels—the temperature profiles on the top and bottom walls are in fact dominated by convection, and not by radiation. Under such conditions the coupling between the radiation and free convection at the top and bottom walls does not significantly affect the radiation heat transfer itself. Regarding the convection heat transfer itself, ElSherbiny et al.[6] have stated that for tall cavities the free convection heat transfer is insensitive to boundary conditions on the top and bottom walls. However, in a related study on square cavities, Balaji and Venkateshan[7] have found that the convection heat transfer could change by as much as 10% in the case of a square cavity, and the radiation heat transfer could change by 3%. Since the present study deals with cavities with $A = 2$ and more, one can expect that the actual errors encountered will be much less compared to the above said values, which by themselves are not high considering the error band associated with correlating a large number of data. Hence, the decoupling of the two modes of heat transfer for the problem under consideration is justified.

Radiation Equations

The 21 × 21 nonuniform grid for convection has been retained for radiation. The view factors are evaluated using the Hottel's cross string method.[8] The entire enclosure is divided into 80 zones with 20 zones on each wall. The total number of view factors is 6400. The radiosity irradiation method has been used for obtaining the radiant fluxes. The appropriate radiosity equations in nondimensional form are

$$j_i = \varepsilon_i(T_i/T_H)^4 + (1 - \varepsilon_i)\sum_{j=1}^{80} F_{ij}j_j \qquad i = 1\text{–}80 \tag{5}$$

The temperature values on the top and bottom walls used in the radiosity equations are obtained by solving Eqs. (1–3). The radiosity equations are solved by using the Gaussian elimination method which is a direct solver. While presenting the results, radiation Nusselt number is normalized with respect to N_{RC} for elegance in presentation, and most importantly, for commonly encountered transparent fluids (optically thin fluids) like air, the N_{RC} is usually of the order of unity. The $\overline{Nu}_R$ defined in the present study when multiplied by N_{RC} gives the radiation Nusselt number which is compatible with the convection Nusselt number, and these two when added give $\overline{Nu}_o$:

$$\overline{Nu}_o = \overline{Nu}_c + N_{RC}\overline{Nu}_R \tag{6}$$

Results and Discussion

Based on the present numerical study a correlation for the mean convection Nusselt number has been obtained as

$$\overline{Nu}_c = 0.215Ra^{0.265}A^{-0.215} \text{ (error band, } \pm 12\%) \tag{7}$$

The above correlation is in excellent agreement with the widely quoted numerical correlation of Berkovsky and Polevikov.[9] Also, the correlation is in very good agreement with the measurements of ElSherbiny et al.[10] Typically, for $Ra = 10^4$ ($A = 4$) and $Ra = 10^5$ ($A = 8$), the difference between the present correlation and that of ElSherbiny et al. is around 6%. This establishes the soundness of the convection code. As regards the radiation heat transfer, Table 1 shows the typical range of parameters considered in the present study. The $\overline{Nu}_R$ is correlated as

$$\overline{Nu}_R = 0.966(2/\varepsilon_H - 1)^{-0.957}(1 - T_R^4)^{1.042}[A/(1 + A)]^{0.129} \tag{8}$$

The above results are for $\varepsilon_c = \varepsilon_H$. The correlation coefficient is 99.97% and the error band is ±3%. Figure 1 shows excellent agreement between the data and the correlation. The form of the correlation has a definite physical basis. Figure 2 shows the radiation Nusselt number as a function of aspect ratio with the other parameters fixed at the values shown in the figure. It is seen that the radiation Nusselt number approaches the value given by the parallel plate formula with increasing aspect ratio, and therefore, at very high aspect ratios the parallel plate formula may be a reasonable approximation. Hence, the aspect ratio effect is accounted for by using the form $A/(1 + A)$, and this value tends to 1 as A tends to infinity. Equation (8) has been correlated from a set of 105 data points covering a wide range of parameters shown in Table 1. Therefore, the apparent scatter between the data and correlation in Fig. 2 is due to the fact that the above figure has been drawn for a set of six points corresponding to a particular temperature ratio (0.7) and emissivity (0.95). The overall goodness of the fit can be very clearly seen in Fig. 1. Regarding the emissivity of the hot wall, it is very clear that if $\varepsilon_H = 0$, $\overline{Nu}_R = 0$, and when $\varepsilon_H = 1$, $\overline{Nu}_R$ is a maximum. Hence the power law form is used for emissivity. Also, the correlation is now in a form similar to the parallel plate formula which is given by

$$\overline{Nu}_R = 1.0(2/\varepsilon_H - 1)^{-1.0}(1 - T_R^4)^{1.0} \tag{9}$$

[The familiar parallel plate formula, $q_{rad} = \sigma(T_H^4 - T_C^4)/(1/\varepsilon_H + 1/\varepsilon_c - 1)$ is recast in the present form for the case of $\varepsilon_H = \varepsilon_c$, to facilitate comparison with Eq. (8).]

The difference in coefficients/exponents between Eqs. (8) and (9), in essence, sum up the effects of convection and the finiteness of the cavity on the radiation heat transfer. The correlation obtained in the present work is valid for $Ra > 500A$ for each of the aspect ratios. The condition $Ra > 500A$ is usually taken as the transition from conduction regime to convection in cavity flows.[11] Figure 3 clearly indicates that Rayleigh number has negligible influence on $\overline{Nu}_R$, and therefore, does not appear in Eq. (8). A fairly wide range of Rayleigh numbers has been used in the calculations, and the ±3% error band in the correlation takes care of the Rayleigh number influence. (For similar reasons ε_{top} and ε_{bot} have been omitted from the correlation.) The importance of temperature profiles of the top and bottom walls in deciding the radiant heat transfer is seen in Fig. 4. The characteristic temperature profiles can be obtained only by a solution of the flow equations. The importance of temperature distribution along the walls in solving such combined problems has been qualitatively stated by ElSherbiny et al.[6]

Table 1 Range of parameters considered in the present study

Range of parameters
$0.2 < Pr < 20$
$2 < A < 20$
$10^3 < Ra < 10^6$
$0.05 < \varepsilon_H = \varepsilon_c < 0.95$
$0.05 < \varepsilon_{top} = \varepsilon_{bot} < 0.95$
$0.6 < T_R < 0.9$

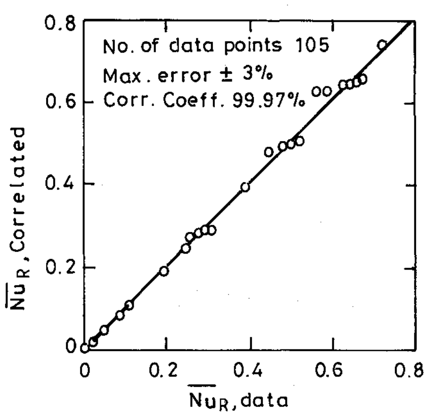

Fig. 1 Comparison of $\overline{Nu}_R$ (data) with $\overline{Nu}_R$ (correlated).

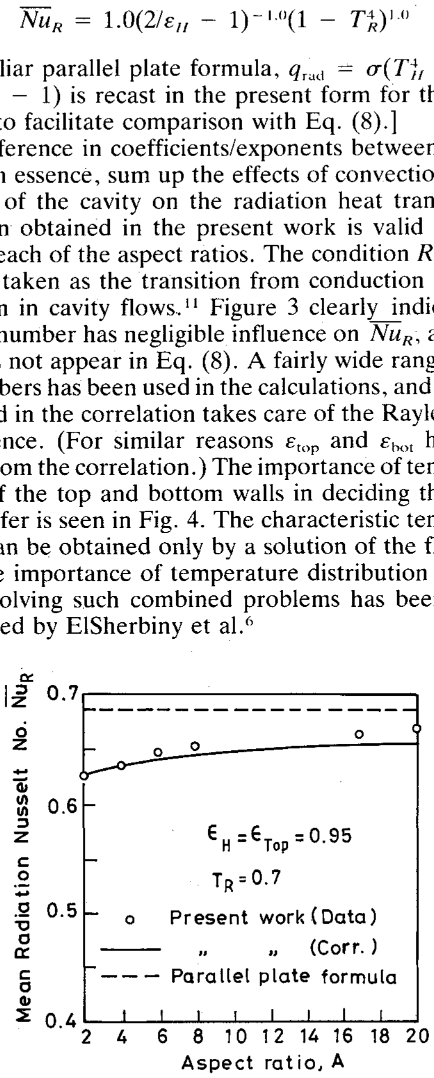

Fig. 2 Effect of aspect ratio on $\overline{Nu}_R$.

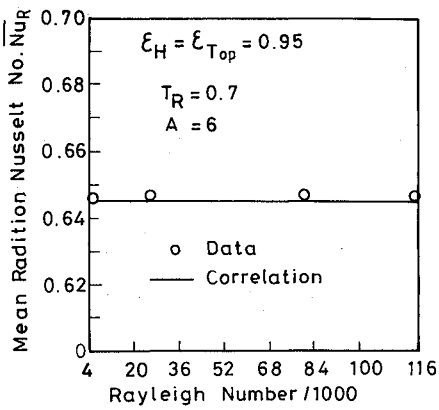

Fig. 3 Effect of Rayleigh number on $\overline{Nu}_R$.

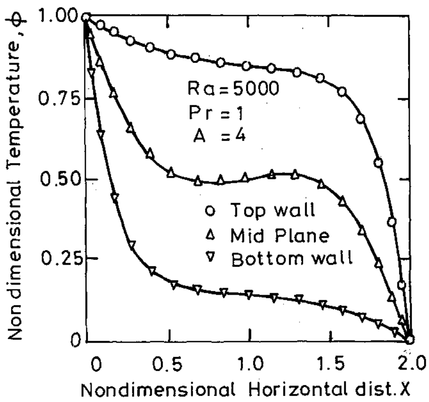

Fig. 4 Temperature profiles across the cavity due to convection.

The difference in $\overline{Nu}_R$ obtained from Eqs. (8) and (9) can be as high as 15%. Since the present correlation covers a wide range of aspect ratios and is mathematically more rigorous, it is recommended as a replacement for the parallel plate formula in the range of parameters shown in Table 1. The present calculations are for the laminar flow regime. At high aspect ratios when the Ra_H exceeds 10^9 or so, the flow may change to the turbulent regime. Hence, correlations for turbulent flow in cavities have to be used in determining the free convection heat transfer. Regarding the radiation, even at high aspect ratios, the present calculations are superior to the parallel plate formula. Therefore, the correlation for the radiation heat transfer can be used to predict the radiant heat flux, but with a caution that a laminar solution is being extrapolated to the turbulent regime as regards the wall temperature distribution, and to that extent the present correlation may be in error. However, at low aspect ratios, but high Ra due to large temperature differences, the temperature profiles on the top and bottom have to be obtained by solving the flow equations which take into account the effect of turbulence. Also at this point it needs to be emphasised that the above calculations are not valid in situations where the fluid flow is three-dimensional. Only if the depth of the cavity is very large compared to the spacing, two-dimensional approximation is reasonable.

Conclusions

Detailed numerical calculations have been done in cavities with A in the range of 2–20 when both free convection and surface radiation are present. The use of temperature profiles obtained by solving the flow equations for determining the heat flux, along with a detailed enclosure analysis, clearly expose the hollowness of the simple parallel plate formula for radiation, which neither considers this effect nor the effect of aspect ratio. At the same time, for cavities with $A \geq 2$, the decoupling of the two mechanisms of heat transfer is possible, and on this basis comprehensive correlations for radiation Nusselt number and convection Nusselt number have been proposed.

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Natural Convection in Horizontal Porous Annuli with Circumferential Baffles

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Introduction

To suppress natural convection within insulation is one of the important considerations in process design. Recently, Bejan[1,2] has shown that using internal partitions can effectively reduce convective heat losses. For pipes, this can be implemented by insertion of radial baffles in the insulation.[3] In this Note, an alternative approach, i.e., using circumferential baffles, is investigated. The same technique has been applied to air-filled annuli by Yang et al.[4] and proved to be successful. In this study, designs that use only one single baffle (cases I and II) are examined first, followed by the design that requires two baffles (case III). For the case of single baffle, two baffle locations are considered: one is directly above the inner cylinder (case I), while the other is underneath it (case II). Numerical results are obtained for a complete set of baffle angular span (0 deg $\leq \delta \leq$ 360 deg) and a range of Rayleigh number of practical interest ($50 \leq Ra \leq 500$).

Formulation and Numerical Method

For most pipe insulation available today, they can be adequately modeled as a porous annulus. For a typical application, the inner surface is heated at a constant temperature T_i, while the outer surface is maintained at the ambient temperature T_0 ($T_i > T_0$). Having invoked the Boussinesq ap-

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